Welcome to the study of Business Finance. The major topic in this module is the Time Value of Money. And this is the first presentation – Present value and Future value.
The Time Value of Money is an important concept at the very foundation of the field of finance. The basic idea is that it’s more valuable if you can receive cash sooner rather than later. If you get the cash quicker, you can put it to work in other investments and make even more money in the meantime. However, we need to find an objective way to measure the value of receiving cash earlier or later so we can make intelligent decisions on when and where to invest our cash.

There are three presentations in this section on the Time Value of Money, each about 30 minutes in length. Let’s look ahead at an outline of this study.
Outline of Presentations on Time Value of Money

1. Present Value and Future Value

In the first presentation, where you are right now, we begin with the basic concepts of the present value and the future value.
Outline of Presentations on Time Value of Money

1. Present Value and Future Value
2. Compounding Less Than One Year

In the second presentation, we look at the special case when there is compounding in less than a one-year period.
In the final presentation, we learn how to value bonds and stocks. Let’s begin this first presentation with an example to illustrate the idea of present values and future values.
Suppose you have $100 and can invest at 6% per year.

How much will you have in 1 year?

Suppose you have $100 and you can invest this money in a savings account that earns 6% per year. How much will you have in 1 year? {pause 4 seconds}

Many of you can already see that the answer is $106. But let’s slow down a bit and go step by step through the calculation.
We begin with the current value, or present value, of $100. Then, we multiply by 6% to find the interest we would earn in one year, which is $6.
Next, we add the $6 interest to the original $100 principal to arrive at a total of $106 in one year. So if you invested $100 in a savings account that earns 6% per year, you would have $106 in one year. Now, how much would you have in 2 years?
Following the same procedure, we multiply the $106 at the end of one year times .06 to find the interest of $6.36 you would earn during the second year.
Adding back the principal of $106 that you had at the beginning of the second year gives you a total of $112.36 at the end of two years.

Okay, how much would you have in 3 years?
Again, we multiply by .06 to find that you would earn $6.74 in interest during the third year.
Adding the beginning principal of $112.36 gives you a total of $119.10 at the end of three years. This method of calculating interest and adding back the principal gives you the right answer every time, but it is a lot of work. Let me show you a few shortcuts in this calculation that will save a lot of effort.
Notice that to get the $106 at the end of one year in the savings account, we added the original principal of $100
to the interest, which is $100 times .06 .
In equation form, this is $100 + $100 (.06)$ . But we can factor out the $100$ and get $100$ times 1 plus the interest rate, or $1.06$ .
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Total in 1 year

Total in 2 years

Total in 3 years

So we can calculate the total amount in the account after one year by simply multiplying by 1 plus the interest rate, and save a lot of work.
We can also repeat this process for the second year by multiplying times 1 plus the interest rate a second time to get $112.36.
And for three years, we can multiply times 1 plus the interest rate a third time to get $119.10 at the end of 3 years.
Notice again that to get the total amount in 3 years, earning 6% per year, we just multiplied by 1.06 three times. There is actually an easier way to do this, all in one step.
We can simply multiply times 1.06 raised to the third power. This is the same thing as multiplying by 1.06 three times.

Let’s briefly look at how you would do this on a calculator. You’ll need a simple calculator that has a $y^x$ key on it in order to follow the example on the next slide. If you are using an HP calculator, you will need to convert to Reverse Polish notation. But most calculators will work as illustrated on the next slide.
First, turn on the calculator, and then enter the initial present value of $100. Then multiply the present value of $100 times 1 plus the interest rate raised to the third power. For this problem, enter 100 into the display, press the multiply key, enter 1.06 into the display, press the y^x key, enter 3 into the display, and press the equal sign key. $100 \times 1.06^{3} = 119.10$. This method of calculating how money compounds each year in a savings account is a real time saver compared to the original calculation of adding the principal to the interest each year. To demonstrate this shortcut, let’s do one more problem. If you deposit $100 in a savings account today, how much will you have in 40 years if the account earns 6% per year?
We can use the same procedure as before, except we raise 1 plus the interest rate to the 40th power. $100 \times 1.06^{40} = 1028.57$. This means that $100$ deposited in a 6% savings account for 40 years will become $1028.57$. This process is much more efficient than the old way of adding back the interest each year, which would take 30 minutes or more of hard work. We can formalize this method of calculating the growth of money, compounded over several years, with a general equation.
The Future Value is equal to the Present Value times 1 plus the interest rate, \((1+r)\), raised to the \(t\) power, where \(t\) is the number of years. This formula just puts the calculation from the previous slide into a general formula. Let’s use this formula to solve a problem.
Suppose your Uncle dies and leaves money to you in his will. He gives you two options: (1) $100,000 now, or (2) $115,000 three years from now. You can invest at 6% per year. Which do you prefer?

Suppose your Uncle dies and leaves money to you in his will. He gives you two options: (1) $100,000 now, or (2) $115,000 three years from now. Suppose also that you can invest at 6% per year. Which do you prefer? Now, to solve this problem, we need to get both options on the same basis. The way the options are presented, the money would be received at different times, so they are not on the same basis. $115,000 is more money in three years, but how much would you have in three years if you took the $100,000 now.
Suppose your Uncle dies and leaves money to you in his will. He gives you two options: (1) $100,000 now, or (2) $115,000 three years from now. You can invest at 6% per year. Which do you prefer?

$100,000 \times (1.06)^3 = $119,100 \text{ in 3 years}

With the first option, $100,000 invested at 6% per year for 3 years would give you $119,100. We can calculate this by multiplying the Present Value of $100,000 times one plus the interest rate, (1.06), raised to the third power, which gives us $119,100. Since this is more than $115,000 in 3 years, you should take the $100,000 now. The $100,000 now is actually worth more money than $115,000 in 3 years. So option 1 is the best deal. This problem can also be solved correctly another way.
Recall earlier that we derived the formula that the Future Value is equal to the Present Value times one plus the interest rate raised to the $t$ power.
We could divide both sides by \((1+r)^t\) raised to the \(t\) power, and this would give us the formula for the Present Value. The Present Value is equal to the Future Value divided by one plus the interest rate raised to the \(t\) power. This is really the same formula, just in a different form. Now we can solve the previous problem a different way. Let’s calculate the Present Value of the $115,000 3 years from now.
The Present Value is equal to the Future Value divided by one plus the interest rate raised to the third power. In this example, the future value is $115,000 divided by 1.06 raised to the third power. This gives us $96,556. This means that if you deposited $96,556 today in a savings account that earns 6% per year, you would have $115,000 in 3 years. So $96,556 today is really the same thing as $115,000 3 years from now.
So now you can compare the Present Value of $100,000 for Option 1 with the Present Value of $96,556 for Option 2, and it’s easy to see that Option 1 is the better deal. This is the same answer we determined earlier using a common time frame of 3 years from now. You will always get the same relative answer if you use a common time frame to compare money at any point in time. But it’s often more convenient to use today as the common time frame. That’s why Present Values are used more often than any particular point of time in the future.

Let’s solve a few more problems to make sure you know how to calculate future values and present values.
If you deposited $300 in a savings account that earns 9% per year, how much will you have in 5 years?
If you deposited $300 in a savings account that earns 9% per year, how much will you have in 5 years?

\[ FV = PV (1+r)^t = $300 \times (1.09)^5 = $461.59 \]

The Future Value is equal to the Present Value times one plus the interest rate raised to the power, $300 \times 1.09^5$ equals $461.59$. So $300$ would become $461.59$ in 5 years if it grew at 9% per year. Let’s work another problem.
Your bank will let you borrow at 10% per year. You know you will have $300,000 in 7 years to pay off a balloon note. How much can you borrow today?

Your bank will let you borrow at 10% per year. You know you will have $300,000 available to you in 7 years to pay off a balloon note. How much can you borrow today? Since the question asks for an amount today, we need to calculate a present value.
Your bank will let you borrow at 10% per year. You know you will have $300,000 in 7 years to pay off a balloon note. How much can you borrow today?

\[
PV = \frac{FV}{(1+r)^t} = \frac{\$300,000}{(1.10)^7} = \$153,947
\]

The Present Value equals the Future Value divided by one plus the interest rate raised to the t power. In this example, the Future Value is $300,000, so $300,000 divided by 1.10 raised to the seventh power equals $153,947. So you could borrow $153,947 today and pay the loan off with a lump sum payment of $300,000 seven years from now. Let’s look at another problem.
You are the executor of your Uncle’s estate. He wants to leave enough money that his grandchild will have $20,000 when he enters college in 15 years. If interest rates are 6%, how much money should be put in a Trust Fund today?

Suppose you are the executor to your Uncle’s estate. He wants to leave enough money that his grandchild will have $20,000 when he enters college in 15 years. If interest rates are 6% per year, how much money should be put into a trust fund today so that it will grow to $20,000 in 15 years?

The first question you should address is whether the problem is asking for a future value or a present value. Since the question refers to how much money we need today, it is pretty clear that we need to calculate the present value.
You are the executor of your Uncle’s estate. He wants to leave enough money that his grandchild will have $20,000 when he enters college in 15 years. If interest rates are 6%, how much money should be put in a Trust Fund today?

\[
PV = \frac{FV}{(1+r)^t}
\]

The Present Value is equal to the Future Value divided by one plus the interest rate raised to the \( t \) power.
You are the executor of your Uncle’s estate. He wants to leave enough money that his grandchild will have $20,000 when he enters college in 15 years. If interest rates are 6%, how much money should be put in a Trust Fund today?

\[
P V = \frac{F V}{(1+r)^t} = \frac{20,000}{(1.06)^{15}} = 8,345.30
\]

In this problem, the Future Value is $20,000. So $20,000 divided by 1.06 raised to the 15th power is equal to $8,345.30.

On most calculators, you can enter $20,000 into the display, press the divide key, enter 1.06, press the y^x key, enter 15, and then the equal sign. $20,000 divided by 1.06 y^x 15 equals $8,345.30. A few calculators don’t use the AOS hierarchy of operations, and don’t give the correct answer the way I have shown. Let me show you another way to calculate the correct answer that works all the time.
You are the executor of your Uncle’s estate. He wants to leave enough money that his grandchild will have $20,000 when he enters college in 15 years. If interest rates are 6%, how much money should be put in a Trust Fund today?

\[ PV = \frac{20,000}{(1.06)^{15}} = \frac{20,000}{1.06^{15}} = \frac{1}{(1.06)^{15}} = 8,345.30 \]

$20,000 divided by 1.06 to the 15th power is the same thing as $20,000 times one over 1.06 to the 15th power. The trick is to always start with the \((1+r)\) and work from there to complete the problem. With your calculator, enter \(1.06 \times 15\) equals. That’s the denominator. Since we need one over that, press the \(1/x\) key. Then multiply by $20,000 to equal $8,345.30. We’ll use this approach again when we deal with annuities a little later.

Let’s work another problem to show that Present Values are additive.
Suppose a friend offers you a business deal in which he promises to pay you $500 in one year, $600 in two years, and $700 in three years. How much would you be willing to pay for this investment today if you can earn 6% per year? In fact, your friend explains it this way. If you add the three numbers together, you will eventually receive $1,800. He will give you this $1,800 over the next three years if you pay him $1,650 right now. Are you interested?

To solve this problem, we first must decide whether we need to find the Present Value or the Future Value. Since your friend wants to be paid right now, then we need to find the present value of this investment to see if it is worth $1,650 right now. There is one complication in this problem, however. There are three future values and we only want a single present value for all three. The way to do this is to find the present value for each cash flow individually, and then add the present values together. The first thing we need to do is to organize the facts in this problem so we can clearly see how to calculate the total present value.
An investment will pay $500 in one year, $600 in two years, and $700 in three years. How much would you be willing to pay for this investment today if you can earn 6% per year?

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This slide shows a time line for 1, 2, and 3 years into the future. Slightly below the time line, I show when each of the cash flows occur. From the problem, we know that $500 is paid at the end of one year, $600 is paid at the end of two years, and $700 is paid at the end of three years.

Now we can calculate the present values.
An investment will pay $500 in one year, $600 in two years, and $700 in three years. How much would you be willing to pay for this investment today if you can earn 6% per year?

First of all, the present value of $500 one year from now is the future value of $500 divided by 1.06, or $471.70.
An investment will pay $500 in one year, $600 in two years, and $700 in three years. How much would you be willing to pay for this investment today if you can earn 6% per year?

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Then, the present value of $600 two years from now is $600 divided by 1.06 squared, which equals $534.00.
An investment will pay $500 in one year, $600 in two years, and $700 in three years. How much would you be willing to pay for this investment today if you can earn 6% per year?

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Finally, the present value of $700 three years from now is $700 divided by 1.06 raised to the third power, which equals $587.73.

Now, since these three present values are all on the basis of the same point in time, today, we can add them together.
An investment will pay $500 in one year, $600 in two years, and $700 in three years. How much would you be willing to pay for this investment today if you can earn 6% per year?

\[
\begin{array}{c|c|c|c}
0 & 1 & 2 & 3 \\
\hline
 & 500 & 600 & 700 \\
\end{array}
\]

\[
\begin{array}{c|c|c|c}
 & \frac{1}{1.06^1} & \frac{1}{1.06^2} & \frac{1}{1.06^3} \\
\hline
471.70 & 534.00 & 587.73 & 1593.43 \\
\end{array}
\]

The total present value, then, is $1593.43. Since this investment is worth less than the $1650 your friend wanted right now, you should decline the offer. Or better still, you can make a smaller counter-offer of say $1550 and make an extra $43 on the deal if he accepts!

The point of this problem is to show you how to find a fair current value of an investment opportunity, or to say it more precisely, to find the present value of an investment. When we have more than one present value, we can simply add them together. Let’s work another problem.
Suppose your uncle leaves you money in his will. He stipulates that you will receive $10,000 for each of the next 3 years. How much money will you have in 3 years if you can earn 8% per year?

Again, the first step is to determine whether we need to find the Present Value or the Future Value. Since the problem asks how much money will you have in 3 years, we need to find the Future Value. The next step is to organize the cash flows on a time line so we can clearly see how to calculate the total future value.
Your uncle leaves you money in his will. He stipulates that you will receive $10,000 for each of the next 3 years. How much money will you have in 3 years if you can earn 8% per year?

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This slide shows that you expect to receive $10,000 at the end of one year, another $10,000 at the end of two years, and a third $10,000 at the end of three years. Now we can calculate the future values.

Starting at the right, what is the future value in three years of the $10,000 you expect to receive in three years? This is the easy one. $10,000 in three years is already the future value in three years.

Now, what is the future value in three years of the $10,000 you expect to receive in two years? Notice first, how long will this $10,000 earn 8% per year? From the time line, you can see that this $10,000 will earn 8% per year for one year.
Your uncle leaves you money in his will. He stipulates that you will receive $10,000 for each of the next 3 years. How much money will you have in 3 years if you can earn 8% per year?

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
10,000 & 10,000 & 10,000 & 10,800 \\
\end{array}
\]

The future value is equal to the present value times one plus the interest rate raised to the time power. So $10,000 in two years times 1 plus the interest rate, or 1.08, raised to the one power equals a future value in three years of $10,800.

What is the future value in three years of the $10,000 you expect to receive in one year. Notice that this $10,000 will earn 8% per year for two years.
Your uncle leaves you money in his will. He stipulates that you will receive $10,000 for each of the next 3 years. How much money will you have in 3 years if you can earn 8% per year?

\[
\begin{array}{c|c|c|c}
0 & 1 & 2 & 3 \\
10,000 & 10,000 & 10,000 \\
\end{array}
\]

\[\times 1.08 \quad \times 1.08^2 \]

\[\rightarrow 10,800 \quad \rightarrow 11,664 \]

So $10,000 \times 1.08$ raised to the second power equals a future value in three years of $11,664$. 


Your uncle leaves you money in his will. He stipulates that you will receive $10,000 for each of the next 3 years. How much money will you have in 3 years if you can earn 8% per year?

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\[
\begin{align*}
10,000 & \times 1.08 = 10,800 \\
10,000 & \times 1.08^2 = 11,664 \\
\text{Total Future Value} & = 32,464
\end{align*}
\]

Since we now have all three cash flows converted to the same point in time (at three years from now), we can add the three future values to get a total future value of $32,464.
Suppose the problem asked for the total present value instead of the total future value. We can compare the process of calculating the future value with the process of calculating the present value.
First, we need to construct a time line, as before -- $10,000 each year for the next three years. We need to calculate the present value of each cash flow, and then add the present values. Look at the first cash flow. To find the present value, we need to divide by one plus \( r \) to the \( t \) power.
In this case, $10,000 divided by 1.08 raised to the one power equals $9,259. To find the present value of the second cash flow, we again divide the cash flow by one plus r to the t power.
$10,000 divided by 1.08 squared equals $8,573.
To find the present value of the third cash flow, $10,000 divided by 1.08 raised to the third power equals $7,938.
Summing the three present values yields a total present value of $25,771. This example problem shows how to use a time line and calculate either a future value or a present value.
The answer is a future value of $32,464 or a present value of $25,771. Suppose you had calculated the present value and then discovered that you should have calculated the future value instead. You could do the problem again from scratch. But there would be an easier way.
Future Value = 32,464
Present Value = 25,771

Future Value = Present Value \cdot (1+r)^t

Since the three $10,000 cash flows are equivalent to a single cash flow today of $25,771 (the present value), we could calculate the future value by multiplying the present value times one plus r to the t power.
Future Value = 32,464
Present Value = 25,771

Future Value = Present Value \( (1+r)^t \)
Future Value = 25,771 \( (1.08)^3 \) = 32,464

In this problem, $25,771 times 1.08 raised to the third power equals $32,464. This problem demonstrates that you can convert present values to future values and vice versa quite easily, even if the total present value or the total future value is a complex mixture of cash flows over time.

There is an easier way, however, to calculate in one step the present value or future value of a series of cash flows if the cash flows are all the same size. Such a series of cash flows is called an annuity.
Annuity -- a constant amount deposited or withdrawn each and every period within a given amount of time. For example, a 10-year $200,000 annuity means $20,000 per year for 10 years.

An annuity is defined as a constant amount deposited or withdrawn each and every period within a given amount of time. For example, a 10-year $200,000 annuity means $20,000 per year for 10 years.
Formulas for annuities:

\[
FVA = C \left[ \frac{(1+r)^t - 1}{r} \right]
\]

\[
PVA = C \left[ \frac{1 - \frac{1}{(1+r)^t}}{r} \right]
\]

As it turns out, there are standardized formulas to find the present value or future value of an annuity. This slide shows two formulas, depending on whether you would like to calculate the future value of an annuity or the present value of an annuity.
Formulas for annuities:

\[ FVA = \$10,000 \left[ \frac{(1.08)^3 - 1}{0.08} \right] = \$32,464 \]

\[ PVA = \$10,000 \left[ \frac{1 - \frac{1}{(1.08)^3}}{0.08} \right] = \$25,771 \]

Substituting the numbers in the example problem, the future value of the annuity in the problem can be calculated directly to be $32,464. Likewise, in one step, the present value of the annuity is equal to $25,771. In the next slide, I’ll demonstrate how to use a standard calculator to find the future value or present value of an annuity.
Using a Calculator to calculate FVA:

\[
FVA = \$10,000 \left[ \frac{(1.08)^3 - 1}{.08} \right] = \$32,464
\]

\[
1.08^3 \times 3 - 1 = \div .08 = \times 10,000 = 32464
\]

As we did earlier, the trick is to begin with the (1+r) and then work outward to complete the calculation. For the Future Value of an annuity in the example problem, begin with “1.08 \( y^x \) 3 =- 1 =.” This is the numerator. “Divided by .08 =.” This is the term in the brackets. “Times 10,000 = 32,464,” which is the future value of the annuity.
Using a Calculator to calculate PVA:

\[
PVA = \frac{10,000 \left[ 1 - \frac{1}{(1.08)^3} \right]}{0.08} = 25,771
\]

\[1.08 \ y^3 = 1/x +/- 1 = \frac{1}{.08} \times 10,000 = 25,771\]

The Present Value of an annuity can be calculated in a similar fashion. “1.08 \ y^3 = 1/x +/- plus 1 =.” This is the numerator. “Divided by .08 =.” This is the term in the brackets. “Times 10,000 = 25,771,” which is the present value of the annuity. Let’s work another problem to test your skills in calculating present and future values.
On the television game show *Wheel of Fortune* several years ago, the daily winner was given a choice of several prizes, including $25,000 cash or a $30,000 annuity that pays $1,000 per year for 30 years. If you can earn 8% per year, which option is better?

On the television game show *Wheel of Fortune* several years ago, the daily winner was given a choice of several prizes, including $25,000 cash or a $30,000 annuity that pays $1,000 per year for 30 years. If you can earn 8% per year, which option is the better choice? The first step is to decide whether the problem requires the calculation of a present value or a future value. Since we are comparing $25,000 now to the current value of a 30-year $30,000 annuity, we need to calculate the present value of the annuity.
On the *Wheel of Fortune* several years ago, the daily winner was given a choice of several prizes, including $25,000 cash or a $30,000 annuity that pays $1,000 per year for 30 years. If you can earn 8% per year, which option is better?

\[
PVA = \frac{1 - \frac{1}{(1.08)^{30}}}{0.08} = $11,258
\]

This slide shows the calculation of the present value of this annuity. On a calculator, this would be calculated as 1.08 \( y^x \) 30 equals 1/x equals +/- plus 1 = divided by .08 = times 1000 equals $11,258. This means that it would be much better to choose the $25,000 cash than the $30,000 annuity. As it turns out, none of the contestants would choose the $30,000 annuity, so they eventually changed the rules. The daily winner now gets a random grand prize if he or she can solve the final puzzle. Let’s work another problem.
You want to buy some lake property. The price of the land is $80,000. Suppose you negotiate a deal to purchase the property with annual payments over the next 10 years, with an interest rate of 7% per year. How much would the annual payment be?

You want to buy some lake property. The price on the land is $80,000. Suppose you don’t have $80,000 readily available, so you negotiate a deal to purchase the property with annual payments over the next ten years, with an interest rate of 7% per year. How much would the payment be each year? Notice that the deal is to give the landowner something equivalent to the $80,000 now. Again we must first decide whether we need to calculate a present value or a future value to solve the problem. Since we are trying to find something equivalent to $80,000 now, we need to calculate a present value. Notice also that the annual payment would be the same amount each and every year for 10 years, so we are dealing with an annuity. We can find the solution by calculating the present value of the annuity. Let’s set up the present value of an annuity formula, and put in all the numbers we know from the problem.
You want to buy some lake property. The price of the land is $80,000. Suppose you negotiate a deal to purchase the property with annual payments over the next 10 years, with an interest rate of 7% per year. How much would the annual payment be?

\[ $80,000 = C \left[ \frac{1 - \frac{1}{(1.07)^{10}}}{.07} \right] = 7.02358 \ C \]

Starting from the left, we know that the present value of this annuity must be $80,000 since the landowner wants something equivalent to a present value of $80,000. C is the constant payment each and every year – that is what we want to calculate. (1+r) is 1.07, t is 10 years, and r is .07. So $80,000 is equal to 7.02358 C.
You want to buy some lake property. The price of the land is $80,000. Suppose you negotiate a deal to purchase the property with annual payments over the next 10 years, with an interest rate of 7% per year. How much would the annual payment be?

\[
\frac{1-\frac{1}{(1.07)^{10}}}{0.07} \times 80,000 = 7.02358 \times C
\]

\[
C = \frac{80,000}{7.02358} = 11,390.20
\]

Dividing both sides by 7.02358 gives us C equal to $11,390.20. So we might be able to cut a deal with the landowner by paying him $11,390.20 each year for the next 10 years. Let’s look at a more complicated problem.
Suppose an investment pays $1,000 for each of the next 4 years and $10,000 after 5 years. If you earn 5% per year, how much is this worth today?

Since we want to find how much this is worth today, we need to calculate a present value. Let’s organize the information in the problem so you can see how to solve it.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

This slide shows a time line with the cash flows as presented in the problem. The solution is to find the present value of each cash flow, and add the present values together to get a total present value for the investment.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

The present value of the first cash flow is 1,000 divided by 1.05, or $952.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

The present value of the second cash flow is $1,000 divided by 1.05 squared, or $907.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

The present value of the third cash flow is $1,000 divided by 1.05 raised to the third power, or $864.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

The present value of the fourth cash flow is 1,000 divided by 1.05 raised to the fourth power, or $823.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

And finally, the present value of the fifth cash flow is 10,000 divided by 1.05 raised to the fifth power, or $7,835.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

Adding the five present values yields a total present value of $11,381. So you would be willing to pay $11,381 for this investment today. This method of finding the present value of each cash flow and adding them together works for almost every problem. But sometimes there is a shortcut that will save some time.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

For example, in this problem look at the cash flows on the time line.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

Notice that the first four cash flows form an annuity of $1,000 each year for 4 years. However, that does not include all of the cash flows.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

There is also a single cash flow of $10,000 at the end of the fifth year. So we could separate the problem into two pieces, and find the present value of a 4-year annuity and add that to the present value of a single cash flow in year 5.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

\[ PVA = \frac{1000 \left(1 - \frac{1}{(1.05)^4}\right)}{0.05} = \$3,546 \]

Let’s calculate the present value of the annuity first. Starting from the left, the Present Value of the Annuity is what we want to calculate. C is the constant payment of $1,000 each year, one plus the interest rate is 1.05, t is equal to 4 years, and r is equal to .05. Solving the equation yields a present value $3,546 for the 4-year annuity.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

\[
PVA = \frac{1000}{0.05} \left[ \frac{1 - (1.05)^4}{1.05} \right] = 3,546
\]

\[
PV = \frac{FV}{(1+r)^t} = \frac{10,000}{(1.05)^5} = 7,835
\]

We need to also find the present value of the single cash flow of $10,000 at the end of year 5. The Present Value is equal to the Future Value of $10,000 divided by one plus the interest rate, (1.05), raised to the 5th power, which yields a present value of $7,835.
Suppose an investment pays $1000 for each of the next 4 years and $10,000 after 5 years. If you can earn 5% per year, how much is this worth today?

\[
PVA = \$1,000 \left(1 - \frac{1}{(1.05)^4}\right) = \$3,546
\]

\[
PV = \frac{FV}{(1+r)^t} = \frac{10,000}{(1.05)^5} = 7,835
\]

Total Present Value = $11,381

This adds up to a total present value of $11,381, which is the same answer we calculated earlier using the sum of all five present values. The point of this problem is to show that there are often several ways to solve a complex problem, and you should use any shortcuts that you can to save some effort. For example, when you are finding the present value of a series of cash flows, you can always find the present value of each cash flow and then add them together. But sometimes you can solve the problem easier if you recognize if there is an annuity in the problem.
Let’s summarize the material we have covered in this module.
Summary

• Derived the formulas for Present Value and Future Value.

First, we derived the formulas for the present value and future value of a single cash flow.
Summary

• Derived the formulas for Present Value and Future Value.

• Applied the formulas for Present Value and Future Value of an annuity.

Then we applied the formulas for the Present Value and Future Value of an annuity.
Summary

• Derived the formulas for Present Value and Future Value.

• Applied the formulas for Present Value and Future Value of an annuity.

• Completed several example problems.

Finally, we completed several examples in which we used present values and future values to solve practical problems.

This first presentation on the Time Value of Money has presented some of the basic concepts. The next presentations develop more advanced applications. But for now, this completes the first presentation on the Time Value of Money.